MATH7901

Analysis

Session 1, In person-scheduled-weekday, North Ryde 2024

School of Mathematical and Physical Sciences

Contents

General Information

Unit convenor and teaching staff
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Credit points
10

Prerequisites
Admission to MRes

Corequisites

Co-badged status

Unit description
This unit provides an advanced introduction to the key areas of research interest in modern analysis. We will study Lebesgue integration, positive Borel measures, and the all important function spaces Lp. Then we will study the elementary Hilbert space theory and Banach space techniques. This will provide familiarity with some of the major theorems which make up the analysis toolbox: Monotone and Dominated Convergence theorems; Fatou's lemma; Egorov's theorem; Lusin's theorem; Radon-Nikodym theorem; Fubini-Tonelli theorems about product measures and integration on product spaces; Uniform Boundedness; Fundamental Theorem of Calculus for Lebesgue Integrals; Minkowski's Inequality; Holder's Inequality; Jensen's Inequality; and Bessel's Inequality.

Important Academic Dates

Information about important academic dates including deadlines for withdrawing from units are available at https://www.mq.edu.au/study/calendar-of-dates

Learning Outcomes

On successful completion of this unit, you will be able to:

ULO1: Demonstrate the abstract setting and the logical arguments in metric spaces, Banach spaces and Hilbert spaces, and formulate the structure on positive Borel measure, and the Lebesgue spaces, including the Riesz representation theorem and the approximation by continuous functions

ULO2: Analyse the abstract setting of metric space, Banach space and Hilbert space,

and evaluate their utility to study particular areas of differential equations and calculus in higher dimension setting.

ULO3: Analyse the structure of Lebesgue spaces, and evaluate the utility of this setting to study particular areas of Fourier series and Fourier transforms, harmonic functions and Poisson integral

ULO4: Create a precise mathematical problem from the specific modelling by applying these abstract settings, and discuss the solution by synthesising the methods in the theory of Banach space and Hilbert space

Assessment Tasks

Name	Weighting	Hurdle	Due
Assignment 1	20%	No	week5
Assignment 2	20%	No	week7
Assignment 3	20%	No	week9
Assignment 4	20%	No	week11
Assignment 5	20%	No	week13

Assignment 1

Assessment Type [1]: Problem set
Indicative Time on Task [2]: 4 hours
Due: **week5**
Weighting: **20%**

Set of questions (with short answers required) in (1) Abstract integration, (2) the theory of metric space, (3) elementary properties of measures and integration

On successful completion you will be able to:

- Demonstrate the abstract setting and the logical arguments in metric spaces, Banach spaces and Hilbert spaces, and formulate the structure on positive Borel measure, and the Lebesgue spaces, including the Riesz representation theorem and the approximation by continuous functions
- Analyse the abstract setting of metric space, Banach space and Hilbert space, and evaluate their utility to study particular areas of differential equations and calculus in higher dimension setting.
- Analyse the structure of Lebesgue spaces, and evaluate the utility of this setting to study

particular areas of Fourier series and Fourier transforms, harmonic functions and Poisson integral

Assignment 2

Assessment Type [1]: Problem set
Indicative Time on Task [2]: 4 hours
Due: **week7**
Weighting: **20%**

Set of questions (with short answers required) in (1) the Riesz representation theorem, (2) properties of Borel functions, (3) continuity properties of measurable functions

On successful completion you will be able to:

- Demonstrate the abstract setting and the logical arguments in metric spaces, Banach spaces and Hilbert spaces, and formulate the structure on positive Borel measure, and the Lebesgue spaces, including the Riesz representation theorem and the approximation by continuous functions
- Analyse the abstract setting of metric space, Banach space and Hilbert space, and evaluate their utility to study particular areas of differential equations and calculus in higher dimension setting.
- Analyse the structure of Lebesgue spaces, and evaluate the utility of this setting to study particular areas of Fourier series and Fourier transforms, harmonic functions and Poisson integral

Assignment 3

Assessment Type [1]: Problem set
Indicative Time on Task [2]: 4 hours
Due: **week9**
Weighting: **20%**

Set of questions (with short answers required) in (1) approximation by continuous functions, (2) convex function and inequalities, (3) Lebesgue space and application.

On successful completion you will be able to:

- Demonstrate the abstract setting and the logical arguments in metric spaces, Banach spaces and Hilbert spaces, and formulate the structure on positive Borel measure, and the Lebesgue spaces, including the Riesz representation theorem and the approximation by continuous functions
- Analyse the abstract setting of metric space, Banach space and Hilbert space, and evaluate their utility to study particular areas of differential equations and calculus in

higher dimension setting.
- Analyse the structure of Lebesgue spaces, and evaluate the utility of this setting to study particular areas of Fourier series and Fourier transforms, harmonic functions and Poisson integral

Assignment 4

Assessment Type [1]: Problem set
Indicative Time on Task [2]: 4 hours
Due: **week11**
Weighting: **20%**

Set of questions (with short answers required) in (1) inner product and linear functionals, (2) orthogonal sets, (3) Fourier series .

On successful completion you will be able to:
- Demonstrate the abstract setting and the logical arguments in metric spaces, Banach spaces and Hilbert spaces, and formulate the structure on positive Borel measure, and the Lebesgue spaces, including the Riesz representation theorem and the approximation by continuous functions
- Analyse the abstract setting of metric space, Banach space and Hilbert space, and evaluate their utility to study particular areas of differential equations and calculus in higher dimension setting.
- Create a precise mathematical problem from the specific modelling by applying these abstract settings, and discuss the solution by synthesising the methods in the theory of Banach space and Hilbert space

Assignment 5

Assessment Type [1]: Problem set
Indicative Time on Task [2]: 4 hours
Due: **week13**
Weighting: **20%**

Set of questions (with short answers required) in (1) Fourier series of continuous functions, (2) Hahn-Banach theorem, (3) Poisson integrals.

On successful completion you will be able to:
- Demonstrate the abstract setting and the logical arguments in metric spaces, Banach spaces and Hilbert spaces, and formulate the structure on positive Borel measure, and the Lebesgue spaces, including the Riesz representation theorem and the approximation by continuous functions

- Analyse the abstract setting of metric space, Banach space and Hilbert space, and evaluate their utility to study particular areas of differential equations and calculus in higher dimension setting.
- Create a precise mathematical problem from the specific modelling by applying these abstract settings, and discuss the solution by synthesising the methods in the theory of Banach space and Hilbert space

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[1] If you need help with your assignment, please contact:

- the academic teaching staff in your unit for guidance in understanding or completing this type of assessment
- the Writing Centre for academic skills support.

[2] Indicative time-on-task is an estimate of the time required for completion of the assessment task and is subject to individual variation

Policies and Procedures

Macquarie University policies and procedures are accessible from Policy Central (https://policies.mq.edu.au). Students should be aware of the following policies in particular with regard to Learning and Teaching:

- Academic Appeals Policy
- Academic Integrity Policy
- Academic Progression Policy
- Assessment Policy
- Fitness to Practice Procedure
- Assessment Procedure
- Complaints Resolution Procedure for Students and Members of the Public
- Special Consideration Policy

Students seeking more policy resources can visit Student Policies (https://students.mq.edu.au/support/study/policies). It is your one-stop-shop for the key policies you need to know about throughout your undergraduate student journey.

To find other policies relating to Teaching and Learning, visit Policy Central (https://policies.mq.edu.au) and use the search tool.

Student Code of Conduct

Macquarie University students have a responsibility to be familiar with the Student Code of Conduct: https://students.mq.edu.au/admin/other-resources/student-conduct

Results

Results published on platform other than eStudent, (eg. iLearn, Coursera etc.) or released directly by your Unit Convenor, are not confirmed as they are subject to final approval by the University. Once approved, final results will be sent to your student email address and will be made available in eStudent. For more information visit ask.mq.edu.au or if you are a Global MBA student contact globalmba.support@mq.edu.au

Academic Integrity

At Macquarie, we believe academic integrity – honesty, respect, trust, responsibility, fairness and courage – is at the core of learning, teaching and research. We recognise that meeting the expectations required to complete your assessments can be challenging. So, we offer you a range of resources and services to help you reach your potential, including free online writing and maths support, academic skills development and wellbeing consultations.

Student Support

Macquarie University provides a range of support services for students. For details, visit http://students.mq.edu.au/support/

The Writing Centre

The Writing Centre provides resources to develop your English language proficiency, academic writing, and communication skills.

- Workshops
- Chat with a WriteWISE peer writing leader
- Access StudyWISE
- Upload an assignment to Studiosity
- Complete the Academic Integrity Module

The Library provides online and face to face support to help you find and use relevant information resources.

- Subject and Research Guides
- Ask a Librarian

Student Services and Support

Macquarie University offers a range of Student Support Services including:

- IT Support
- Accessibility and disability support with study
- Mental health support
- Safety support to respond to bullying, harassment, sexual harassment and sexual assault

- Social support including information about finances, tenancy and legal issues
- Student Advocacy provides independent advice on MQ policies, procedures, and processes

Student Enquiries

Got a question? Ask us via AskMQ, or contact Service Connect.

IT Help

For help with University computer systems and technology, visit http://www.mq.edu.au/about_us/offices_and_units/information_technology/help/.

When using the University's IT, you must adhere to the Acceptable Use of IT Resources Policy. The policy applies to all who connect to the MQ network including students.

Unit information based on version 2024.01R of the Handbook