

# **STAT272**

# **Probability**

S1 Day 2016

Dept of Statistics

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#### Disclaimer

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# **General Information**

Unit convenor and teaching staff

**Unit Convenor** 

Georgy Sofronov

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Contact via georgy.sofronov@mq.edu.au

Room 2.362 Level 2 Australian Hearing Hub

Wednesday 12-2pm

Lecturer

Justin Wishart

#### justin.wishart@mq.edu.au

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Level 2, Australian Hearing Hub

Monday 2-4pm

Credit points

3

#### Prerequisites

STAT171(P) and (MATH133(P) or MATH136(P))

Corequisites

#### Co-badged status

#### Unit description

This unit is a mathematically-based introduction to probability theory. Emphasis is placed on the theoretical development of the subject matter. Students should be mathematically competent, especially in the areas of integration, differentiation and the summation of infinite series. Students who are not confident about their ability in these areas should consider enrolling in the more general unit, STAT273. Topics include: conditional probability; discrete and continuous random variables; transformations; convolutions; moments and moment generating functions; central limit theorem; sampling distributions; order statistics; and bivariate distributions.

# Important Academic Dates

Information about important academic dates including deadlines for withdrawing from units are available at <a href="https://www.mq.edu.au/study/calendar-of-dates">https://www.mq.edu.au/study/calendar-of-dates</a>

# **Learning Outcomes**

On successful completion of this unit, you will be able to:

This unit is an introduction to the mathematical foundations of the theory of probability, and thus provides the basic mathematical techniques needed for the theory of statistics.

By the end of this unit, students will have an understanding of the foundation concepts in probability, including conditional probability, random variables and discrete and continuous probability distributions

Students will be able to derive key charateristics of probability distributions, including moments and moment generating functions.

Students will have an understanding of the distributions of sums of independent random variables, the Central Limit Theorem and convolutions.

Students will also understand and be able to derive the distributions of transformed random variables, order statistics, compound random variables and multivariate random variables.

### **Assessment Tasks**

Name	Weighting	Due
Test	10%	Week 6
Assignments	30%	Week 4, 9, 12
Final examination	60%	University Examination Period

### Test

Due: Week 6
Weighting: 10%

There will be a mid-semester test of 50 minutes duration held during the first lecture of week 6. Students are permitted to take in to the test one sheet of A4 paper containing the student's personal summary. One or both sides of the sheet may be used. The material thereon may be in the student's own handwriting (scanned copies are not permitted) and not typed.

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# **Assignments**

Due: **Week 4, 9, 12** Weighting: **30%** 

There will be three assignments, the first one due in week 4, Assignment 2 due in week 9 and Assignment 3 due in week 12. On-time submission of all assignments is compulsory. Late submission of assignments will not be accepted without a good reason, and extension requests should be made directly to the Unit Convenor.

Assignment submission: Assignments are to be submitted to your tutor, in your tutorial in the week in which they are due. No extensions will be considered unless satisfactory documentation outlining illness or misadventure is submitted.

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### Final examination

Due: University Examination Period

Weighting: 60%

The duration of the final examination is three hours plus ten minutes' reading time. An electronic calculator and two A4 sheets of paper (written on one or both sides) may be taken in to the exam room. All material thereon must be in the student's own handwriting (scanned copies are not permitted) and not typed.

For a passing grade, satisfactory performance is required on both: (i) the average of the assignments and test; (ii) the final examination.

You are expected to present yourself for examination at the time and place designated in the University examination timetable, which will be available at https://timetables.mq.edu.au .

Only documented illness or unavoidable disruption may be used as reasons for not sitting an examination at the designated time. In these circumstances you may wish to consider applying for Disruption to Studies via ask.mq.edu.au. Information about the Disruption to Studies policy and procedure is available at: http://students.mq.edu.au/student\_admin/exams/disruption\_to\_studies/

http://www.mq.edu.au/policy/docs/disruption studies/policy.html

http://www.mq.edu.au/policy/docs/disruption\_studies/procedure.html

It is Macquarie University policy not to set early examinations for individuals or groups of students. All students are expected to ensure that they are available until the end of the teaching semester, that is, the final day of the official examination period.

On successful completion you will be able to:

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- Students will also understand and be able to derive the distributions of transformed random variables, order statistics, compound random variables and multivariate random variables.

# **Delivery and Resources**

# Technologies used and required

All unit materials, including administrative updates, lecture notes, tutorials and assignments, will be posted on the Unit website on iLearn. The web address is https://ilearn.mq.edu.au.

Students will attend three one-hour lectures and one one-hour tutorial per week. The notes shown in lectures will be available on iLearn before the lecture is given, but note that corrections may be made after the lecture. Tutorial exercises will be set weekly and will be available on iLearn before the tutorial. Students are expected to have attempted all questions before the tutorial. A plan of the topics to be covered is at the end of this document.

# Required and Recommended texts and/or materials

There is no required textbook for this unit. Students may benefit from having access to the following background reference for additional reading and problems:

"Mathematical Statistics with Applications" W Mendenhall, D Wackerly and R Scheaffer (6th or 7th edition) - library call number is QA276.M426.

The following books may also be useful background references:

ROSS, S. A First Course in Probability (QA273.R83)

SCHEAFFER, R. L. Introduction to Probability and Its Applications (QA273.S357)

SMITH, P. J. Into Statistics (QA276.S615)

FREUND, J. E. Mathematical Statistics (QA276.F692)

HOEL, P. Introduction to Mathematical Statistics (QA276.H57)

HOGG, R.V. & TANIS, E.A. Probability and Statistical Inference (QA273.H694)

LARSON, H. Introduction to Probability Theory and Statistical Inference (QA273.L352)

SPIEGEL, M.R., SRINIVASAN, J. & SCHILLER, J.J. Schaum's outline of theory and problems of probability and statistics (QA273.25.S64)

WALPOLE, R.E. & MYERS, R.H. Probability and Statistics for Engineers and Scientists (TA340.W35)

HOGG, R.V. & CRAIG, A.T. Introduction to Mathematical Statistics (QA276.H59)

At least one copy of each of these is available in the Library, and extra copies may be available on the shelves for borrowing purposes.

It should be understood that there are variations in notation (and even in definition) from one reference book to another, and that the lecture material alone defines recommended notation. Note that all lecture notes will be available in pdf form on the Unit website on iLearn before the lecture. You are required to print out your own copy and bring this to lectures.

# Changes since the last offering of this unit

None.

# **Unit Schedule**

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Sample space, events. Axioms of probability, conditional probability. Bayes Theorem. Random variables and probability distributions.  Discrete Distributions and their applications (Bernoulli, geometric, negative binomial, binomial, hypergeometric, multinomial). The Poisson process and the Poisson distribution.  Continuous random variables and distributions with applications (uniform, exponential, triangular, normal, gamma, beta etc.). Discrete and continuous cumulative distribution functions.  Expected values (discrete and continuous) and properties of the expectation operator. Measures of variation.  Moments: raw and central. Interpretation of moments (skewness, kurtosis etc.).  Sums of independent random variables. Discrete and continuous convolutions with applications.  Distribution of functions (monotonic and non-monotonic) of continuous random variables. Transformation of a continuous random variable to one with a uniform distribution, with applications to simulation.  Probability generating functions and moment generating functions (raw and central) with properties and applications. The moment generating function of a sum of independent random variables. The uniqueness theorem. Characteristic functions.  Chebyshev's inequality. The central limit theorem and applications.  Multivariate (particularly bivariate) random variable theory (continuous and discrete). Marginal and conditional distributions and expectations. Covariance and correlation. Compound distributions.		
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# **Policies and Procedures**

Macquarie University policies and procedures are accessible from <u>Policy Central</u>. Students should be aware of the following policies in particular with regard to Learning and Teaching:

Academic Honesty Policy http://mq.edu.au/policy/docs/academic\_honesty/policy.html

New Assessment Policy in effect from Session 2 2016 http://mq.edu.au/policy/docs/assessment/policy\_2016.html. For more information visit http://students.mq.edu.au/events/2016/07/19/new\_assessment\_policy\_in\_place\_from\_session\_2/

Assessment Policy prior to Session 2 2016 http://mq.edu.au/policy/docs/assessment/policy.html

Grading Policy prior to Session 2 2016 http://mq.edu.au/policy/docs/grading/policy.html

Grade Appeal Policy http://mq.edu.au/policy/docs/gradeappeal/policy.html

Complaint Management Procedure for Students and Members of the Public <a href="http://www.mq.edu.a">http://www.mq.edu.a</a> u/policy/docs/complaint\_management/procedure.html

Disruption to Studies Policy <a href="http://www.mq.edu.au/policy/docs/disruption\_studies/policy.html">http://www.mq.edu.au/policy/docs/disruption\_studies/policy.html</a> The Disruption to Studies Policy is effective from March 3 2014 and replaces the Special Consideration Policy.

In addition, a number of other policies can be found in the <u>Learning and Teaching Category</u> of Policy Central.

#### Student Code of Conduct

Macquarie University students have a responsibility to be familiar with the Student Code of Conduct: https://students.mq.edu.au/support/student\_conduct/

#### Results

Results shown in *iLearn*, or released directly by your Unit Convenor, are not confirmed as they are subject to final approval by the University. Once approved, final results will be sent to your student email address and will be made available in <a href="extraction-color: blue} eStudent</a>. For more information visit <a href="extraction-color: blue} ask.m</a> <a href="equation-color: blue} q.edu.au.

# Student Support

Macquarie University provides a range of support services for students. For details, visit <a href="http://students.mq.edu.au/support/">http://students.mq.edu.au/support/</a>

### **Learning Skills**

Learning Skills (mq.edu.au/learningskills) provides academic writing resources and study strategies to improve your marks and take control of your study.

- Workshops
- StudyWise
- Academic Integrity Module for Students
- Ask a Learning Adviser

# Student Services and Support

Students with a disability are encouraged to contact the <u>Disability Service</u> who can provide appropriate help with any issues that arise during their studies.

# Student Enquiries

For all student enquiries, visit Student Connect at ask.mq.edu.au

# IT Help

For help with University computer systems and technology, visit <a href="http://www.mq.edu.au/about\_us/">http://www.mq.edu.au/about\_us/</a> offices\_and\_units/information\_technology/help/.

When using the University's IT, you must adhere to the <u>Acceptable Use of IT Resources Policy</u>. The policy applies to all who connect to the MQ network including students.

# **Graduate Capabilities**

### Creative and Innovative

Our graduates will also be capable of creative thinking and of creating knowledge. They will be

imaginative and open to experience and capable of innovation at work and in the community. We want them to be engaged in applying their critical, creative thinking.

This graduate capability is supported by:

### Learning outcomes

- This unit is an introduction to the mathematical foundations of the theory of probability, and thus provides the basic mathematical techniques needed for the theory of statistics.
   By the end of this unit, students will have an understanding of the foundation concepts in probability, including conditional probability, random variables and discrete and continuous probability distributions
- Students will be able to derive key charateristics of probability distributions, including moments and moment generating functions.
- Students will have an understanding of the distributions of sums of independent random variables, the Central Limit Theorem and convolutions.
- Students will also understand and be able to derive the distributions of transformed random variables, order statistics, compound random variables and multivariate random variables.

#### Assessment tasks

- Test
- Assignments
- · Final examination

# Discipline Specific Knowledge and Skills

Our graduates will take with them the intellectual development, depth and breadth of knowledge, scholarly understanding, and specific subject content in their chosen fields to make them competent and confident in their subject or profession. They will be able to demonstrate, where relevant, professional technical competence and meet professional standards. They will be able to articulate the structure of knowledge of their discipline, be able to adapt discipline-specific knowledge to novel situations, and be able to contribute from their discipline to inter-disciplinary solutions to problems.

This graduate capability is supported by:

# **Learning outcomes**

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# Critical, Analytical and Integrative Thinking

We want our graduates to be capable of reasoning, questioning and analysing, and to integrate and synthesise learning and knowledge from a range of sources and environments; to be able to critique constraints, assumptions and limitations; to be able to think independently and systemically in relation to scholarly activity, in the workplace, and in the world. We want them to have a level of scientific and information technology literacy.

This graduate capability is supported by:

# Learning outcomes

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#### Assessment tasks

- Test
- Assignments

Final examination

# **Problem Solving and Research Capability**

Our graduates should be capable of researching; of analysing, and interpreting and assessing data and information in various forms; of drawing connections across fields of knowledge; and they should be able to relate their knowledge to complex situations at work or in the world, in order to diagnose and solve problems. We want them to have the confidence to take the initiative in doing so, within an awareness of their own limitations.

This graduate capability is supported by:

### Learning outcomes

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