

# **MATH7901**

# **Analysis**

Session 1, In person-scheduled-weekday, North Ryde 2022

School of Mathematical and Physical Sciences

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#### Disclaimer

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#### **General Information**

Unit convenor and teaching staff

Ji Li

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Credit points

10

Prerequisites

Admission to MRes

Corequisites

Co-badged status

Unit description

This unit provides an advanced introduction to the key areas of research interest in modern analysis. We will study Lebesgue integration, positive Borel measures, and the all important function spaces Lp. Then we will study the elementary Hilbert space theory and Banach space techniques. This will provide familiarity with some of the major theorems which make up the analysis toolbox: Monotone and Dominated Convergence theorems; Fatou's lemma; Egorov's theorem; Lusin's theorem; Radon-Nikodym theorem; Fubini-Tonelli theorems about product measures and integration on product spaces; Uniform Boundedness; Fundamental Theorem of Calculus for Lebesgue Integrals; Minkowski's Inequality; Holder's Inequality; Jensen's Inequality; and Bessel's Inequality.

### Important Academic Dates

Information about important academic dates including deadlines for withdrawing from units are available at <a href="https://www.mq.edu.au/study/calendar-of-dates">https://www.mq.edu.au/study/calendar-of-dates</a>

## **Learning Outcomes**

On successful completion of this unit, you will be able to:

**ULO1:** Demonstrate the abstract setting and the logical arguments in metric spaces, Banach spaces and Hilbert spaces, and formulate the structure on positive Borel measure, and the Lebesgue spaces, including the Riesz representation theorem and the approximation by continuous functions

**ULO2:** Analyse the abstract setting of metric space, Banach space and Hilbert space, and evaluate their utility to study particular areas of differential equations and calculus in higher dimension setting.

**ULO3:** Analyse the structure of Lebesgue spaces, and evaluate the utility of this setting to study particular areas of Fourier series and Fourier transforms, harmonic functions and Poisson integral

**ULO4:** Create a precise mathematical problem from the specific modelling by applying these abstract settings, and discuss the solution by synthesising the methods in the theory of Banach space and Hilbert space

#### **General Assessment Information**

**ATTENDANCE and PARTICIPATION:** Please contact the unit convenor as soon as possible if you have difficulty attending and participating in any classes. There may be alternatives available to make up the work. If there are circumstances that mean you will miss a class, you can apply for Special Consideration via ask.mq.edu.au.

**ASSIGNMENT SUBMISSION**: Assignment submission will be online through the iLearn page.

Submit assignments online via the appropriate assignment link on the iLearn page. A personalised cover sheet is not required with online submissions. Read the submission statement carefully before accepting it as there are substantial penalties for making a false declaration.

- Assignment submission is via iLearn. You should upload this as a single scanned PDF file.
- Please note the quick guide on how to upload your assignments provided on the iLearn page.
- Please make sure that each page in your uploaded assignment corresponds to only one A4 page (do not upload an A3 page worth of content as an A4 page in landscape). If you are using an app like Clear Scanner, please make sure that the photos you are using are clear and shadow-free.
- It is your responsibility to make sure your assignment submission is legible.
- If there are technical obstructions to your submitting online, please email us to let us know.

You may submit as often as required prior to the due date/time. Please note that each submission will completely replace any previous submissions. It is in your interests to make frequent submissions of your partially completed work as insurance against technical or other problems near the submission deadline.

LATE SUBMISSION OF WORK: All assessment tasks must be submitted by the official due date and time. In the case of a late submission for a non-timed assessment (e.g. an assignment), if special consideration has NOT been granted, 20% of the earned mark will be deducted for each 24-hour period (or part thereof) that the submission is late for the first 2 days (including weekends and/or public holidays). For example, if an assignment is submitted 25 hours late, its mark will attract a penalty equal to 40% of the earned mark. After 2 days (including weekends

and public holidays) a mark of 0% will be awarded. Timed assessment tasks (e.g. tests, examinations) do not fall under these rules.

#### **Assessment Tasks**

Name	Weighting	Hurdle	Due
Assignment 1	20%	No	Week 5
Assignment 2	20%	No	Week 7
Assignment 3	20%	No	Week 9
Assignment 4	20%	No	Week 11
Assignment 5	20%	No	Week 13

### **Assignment 1**

Assessment Type 1: Problem set Indicative Time on Task 2: 4 hours

Due: Week 5 Weighting: 20%

Set of questions (with short answers required) in (1) Abstract integration, (2) the theory of metric space, (3) elementary properties of measures and integration

On successful completion you will be able to:

- Demonstrate the abstract setting and the logical arguments in metric spaces, Banach spaces and Hilbert spaces, and formulate the structure on positive Borel measure, and the Lebesgue spaces, including the Riesz representation theorem and the approximation by continuous functions
- Analyse the abstract setting of metric space, Banach space and Hilbert space, and evaluate their utility to study particular areas of differential equations and calculus in higher dimension setting.
- Analyse the structure of Lebesgue spaces, and evaluate the utility of this setting to study particular areas of Fourier series and Fourier transforms, harmonic functions and Poisson integral

#### **Assignment 2**

Assessment Type 1: Problem set Indicative Time on Task 2: 4 hours

Due: Week 7

Weighting: 20%

Set of questions (with short answers required) in (1) the Riesz representation theorem, (2) properties of Borel functions, (3) continuity properties of measurable functions

On successful completion you will be able to:

- Demonstrate the abstract setting and the logical arguments in metric spaces, Banach spaces and Hilbert spaces, and formulate the structure on positive Borel measure, and the Lebesgue spaces, including the Riesz representation theorem and the approximation by continuous functions
- Analyse the abstract setting of metric space, Banach space and Hilbert space, and evaluate their utility to study particular areas of differential equations and calculus in higher dimension setting.
- Analyse the structure of Lebesgue spaces, and evaluate the utility of this setting to study particular areas of Fourier series and Fourier transforms, harmonic functions and Poisson integral

### **Assignment 3**

Assessment Type 1: Problem set Indicative Time on Task 2: 4 hours

Due: Week 9 Weighting: 20%

Set of questions (with short answers required) in (1) approximation by continuous functions, (2) convex function and inequalities, (3) Lebesgue space and application.

On successful completion you will be able to:

- Demonstrate the abstract setting and the logical arguments in metric spaces, Banach spaces and Hilbert spaces, and formulate the structure on positive Borel measure, and the Lebesgue spaces, including the Riesz representation theorem and the approximation by continuous functions
- Analyse the abstract setting of metric space, Banach space and Hilbert space, and evaluate their utility to study particular areas of differential equations and calculus in higher dimension setting.
- Analyse the structure of Lebesgue spaces, and evaluate the utility of this setting to study particular areas of Fourier series and Fourier transforms, harmonic functions and Poisson integral

#### Assignment 4

Assessment Type 1: Problem set Indicative Time on Task 2: 4 hours

Due: Week 11 Weighting: 20%

Set of questions (with short answers required) in (1) inner product and linear functionals, (2) orthogonal sets, (3) Fourier series .

On successful completion you will be able to:

- Demonstrate the abstract setting and the logical arguments in metric spaces, Banach spaces and Hilbert spaces, and formulate the structure on positive Borel measure, and the Lebesgue spaces, including the Riesz representation theorem and the approximation by continuous functions
- Analyse the abstract setting of metric space, Banach space and Hilbert space, and evaluate their utility to study particular areas of differential equations and calculus in higher dimension setting.
- Create a precise mathematical problem from the specific modelling by applying these abstract settings, and discuss the solution by synthesising the methods in the theory of Banach space and Hilbert space

### Assignment 5

Assessment Type 1: Problem set Indicative Time on Task 2: 4 hours

Due: Week 13 Weighting: 20%

Set of questions (with short answers required) in (1) Fourier series of continuous functions, (2) Hahn-Banach theorem, (3) Poisson integrals.

On successful completion you will be able to:

- Demonstrate the abstract setting and the logical arguments in metric spaces, Banach spaces and Hilbert spaces, and formulate the structure on positive Borel measure, and the Lebesgue spaces, including the Riesz representation theorem and the approximation by continuous functions
- Analyse the abstract setting of metric space, Banach space and Hilbert space, and evaluate their utility to study particular areas of differential equations and calculus in higher dimension setting.
- Create a precise mathematical problem from the specific modelling by applying these

abstract settings, and discuss the solution by synthesising the methods in the theory of Banach space and Hilbert space

- the academic teaching staff in your unit for guidance in understanding or completing this type of assessment
- · the Writing Centre for academic skills support.

# **Delivery and Resources**

Related materials: W. Rudin's "Real and complex analysis"

#### **Unit Schedule**

The first five chapters of W. Rudin's book "Real and complex analysis":

Chapter 1: Abstract Integration

Chapter 2: Positive Borel measures

Chapter 3: L<sup>p</sup> spaces

Chapter 4: Elementary Hilbert space theory

Chapter 5: Examples of Banach space techniques

Each of Chapters 1, 2, 4 and 5 takes an average of 5 hours lecturing, and Chapter 3 takes 4 hours.

The last week is for any unforseen delay.

#### **Policies and Procedures**

Macquarie University policies and procedures are accessible from Policy Central (https://policies.mq.edu.au). Students should be aware of the following policies in particular with regard to Learning and Teaching:

- Academic Appeals Policy
- Academic Integrity Policy
- Academic Progression Policy
- Assessment Policy
- · Fitness to Practice Procedure
- Assessment Procedure
- Complaints Resolution Procedure for Students and Members of the Public

<sup>&</sup>lt;sup>1</sup> If you need help with your assignment, please contact:

<sup>&</sup>lt;sup>2</sup> Indicative time-on-task is an estimate of the time required for completion of the assessment task and is subject to individual variation

Special Consideration Policy

Students seeking more policy resources can visit <u>Student Policies</u> (<u>https://students.mq.edu.au/support/study/policies</u>). It is your one-stop-shop for the key policies you need to know about throughout your undergraduate student journey.

To find other policies relating to Teaching and Learning, visit Policy Central (https://policies.mq.e du.au) and use the search tool.

#### Student Code of Conduct

Macquarie University students have a responsibility to be familiar with the Student Code of Conduct: https://students.mq.edu.au/admin/other-resources/student-conduct

#### Results

Results published on platform other than <u>eStudent</u>, (eg. iLearn, Coursera etc.) or released directly by your Unit Convenor, are not confirmed as they are subject to final approval by the University. Once approved, final results will be sent to your student email address and will be made available in <u>eStudent</u>. For more information visit <u>ask.mq.edu.au</u> or if you are a Global MBA student contact <u>globalmba.support@mq.edu.au</u>

## Academic Integrity

At Macquarie, we believe <u>academic integrity</u> – honesty, respect, trust, responsibility, fairness and courage – is at the core of learning, teaching and research. We recognise that meeting the expectations required to complete your assessments can be challenging. So, we offer you a range of resources and services to help you reach your potential, including free <u>online writing and</u> d maths support, academic skills development and wellbeing consultations.

### Student Support

Macquarie University provides a range of support services for students. For details, visit <a href="http://students.mq.edu.au/support/">http://students.mq.edu.au/support/</a>

#### **The Writing Centre**

The Writing Centre provides resources to develop your English language proficiency, academic writing, and communication skills.

- Workshops
- Chat with a WriteWISE peer writing leader
- Access StudyWISE
- Upload an assignment to Studiosity
- Complete the Academic Integrity Module

The Library provides online and face to face support to help you find and use relevant information resources.

Subject and Research Guides

Ask a Librarian

# Student Services and Support

Macquarie University offers a range of **Student Support Services** including:

- IT Support
- Accessibility and disability support with study
- Mental health support
- <u>Safety support</u> to respond to bullying, harassment, sexual harassment and sexual assault
- · Social support including information about finances, tenancy and legal issues

# **Student Enquiries**

Got a question? Ask us via AskMQ, or contact Service Connect.

#### IT Help

For help with University computer systems and technology, visit <a href="http://www.mq.edu.au/about\_us/">http://www.mq.edu.au/about\_us/</a> offices\_and\_units/information\_technology/help/.

When using the University's IT, you must adhere to the <u>Acceptable Use of IT Resources Policy</u>. The policy applies to all who connect to the MQ network including students.